Portfolio Management

Concept ① Types of Returns

ⓐ Holding Period Return
ⓑ Average Return
ⓒ Geometric Average (Geometric Return)
ⓓ Return when Probability is given.

ⓐ Holding Period Return. → The return for period for which you have held the asset.

$$HPR = \frac{D_1 + (P_1 - P_0)}{P_0} \times 100$$

ⓑ Average Return.

When Returns are already given

Year	Return
2026	10%
2027	12%
2028	16%
2029	20%

Average Return = $\frac{\text{Sum of Returns}}{\text{No of Return}}$

$= \frac{10+12+16+20}{4} = 14.5\%$

When we have to compute return also then Average Return.

$Return = \left(\frac{D_1 + P_1 - P_0}{P_0}\right) \times 100$

Year	MPS	DIV	
2026	100	2	
2027	105	2.5	$= \left(\frac{2.5 + 105 - 100}{100}\right) \times 100 = 7.5\%$
2028	98	3	$= \left(\frac{3 + 98 - 105}{105}\right) \times 100 = -3.81\%$
2029	128	3	$= \left(\frac{3 + 128 - 98}{98}\right) \times 100 = 33.67\%$

Average Return = $\frac{7.5\% + (-3.81\%) + 33.67\%}{3}$

$= \boxed{12.45\%}$

Note: First year's Return cannot be calculated.

(C) $\boxed{\text{Geometric Return.}}$

Year 1	Year 2	Year 3
r_1	r_2	r_3

Geometric Average $= \left[(1+r_1)(1+r_2) \ldots (1+r_n)\right]^{\frac{1}{n}} - 1$

or

$= \sqrt[n]{(1+r_1)(1+r_2) \ldots (1+r_n)} - 1$

Self Note:

How to find fractional Power $\Rightarrow x^{\frac{1}{n}}$

Step ① Type x
Step ② Press '$\sqrt{\ }$' 12 Times
Step ③ $-1 =$
Step ④ $\div n =$
Step ⑤ $+1 =$
Step ⑥ '$\times \; =$' 12 Times

ex: $(2.734)^{\frac{1}{5}}$

$= 1.222836$

① Calculating Average Return when Probability is given for different Prices.

2026 — 2027

$P_0 = ₹1000$

P_1	Prob.
$= ₹1100$	0.5
₹1500	0.3
₹1600	0.2

Possibility | Returns × Probability

$$R_i = \left(\frac{D_1 + P_1 - P_0}{P_0}\right) \times 100$$

Possibility 1 $= \left(\frac{1100 - 1000}{1000}\right) \times 100 \times 0.5 = 5\%$

Possibility 2 $= \left(\frac{1500 - 1000}{1000}\right) \times 100 \times 0.3 = 15\%$

Possibility 3 $= \left(\frac{1600 - 1000}{1000}\right) \times 100 \times 0.2 = 12\%$

Average Return. 32%

	A ✗	B ✓
Risk $= \sigma$	$= 2$	$= 3$
Return $= \bar{x} =$	10%	16%
$CoV = \frac{\sigma}{\bar{x}} =$	$\frac{2}{10\%}$	$\frac{3}{16\%}$
	$\Rightarrow \frac{.2}{1\%}$	$\frac{0.1875}{1\%}$

B ✓

Concept ② Risk of Individual Security

- Variance σ^2
- Standard deviation σ
- Coefficient of Variation = $COV = \frac{\sigma}{\bar{x}} \times 100$

Data given in question

Past data given (Ex-post data)

Year	Return $\boxed{X}$	$(x-\bar{x})^2$
2xx1	5%	$(5-\bar{x})^2$
2xx2	10%	$(10-\bar{x})^2$
2xx3	15%	$(15-\bar{x})^2$
2xx4	18%	$(18-\bar{x})^2$
	$\Sigma x =$	$\Sigma(x-\bar{x})^2$

Step ① Mean $= \frac{\Sigma x}{n} = \bar{x}$

Average
expected value

Step ② Calculate $(\text{deviation})^2$
$(x-\bar{x})^2$

Step ③ Variance $= \frac{\Sigma(x-\bar{x})^2}{n}$

Step ④ Standard deviation $\boxed{\sigma} = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

Step ⑤ Coefficient of Variation $= COV = \frac{\sigma}{\bar{x}} \times 100$

$$COV = \frac{\text{Standard deviation}}{\text{Mean}}$$ ← Risk Per unit of Return.

Future predicted data (Ex-ante data) with Probability

x_i Return	P_i Probability	$x_i P_i$	$(x-\bar{x})^2 P_i$
a%	0·3	✓	✓
b%	0·5	✓	✓
c%	0·1	✓	✓
d%	0·1	✓	✓
	1	$\Sigma x_i P_i$ ($\bar{x}$)	$\Sigma(x-\bar{x})^2 P_i$

Step ① Mean $= \bar{x} = \Sigma x_i P_i$

Step ② Calculate $(x-\bar{x})^2 P_i$

Step ③
Variance $= \sigma^2 = \Sigma(x-\bar{x})^2 P_i$

Step ④
Standard deviation $= \sigma = \sqrt{\text{Variance}}$

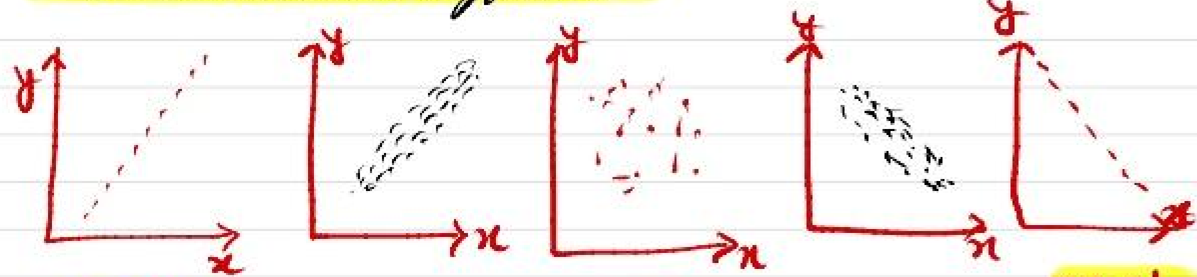
Step ⑤ $COV = \frac{\sigma}{\bar{x}} \times 100$

Concept ③ Correlation

⇒ Correlation coefficient explains relationship between two variables.

ex: Sale of Refrigerators has direct positive correlation with high temp.

r = Correlation Coefficient.



$r = 1$	$0 < r < 1$	$r = 0$	$-1 < r < 0$	$r = -1$
Perfect Positive Correlation	Positive Correlation	No Correlation	Negative Correlation	Perfect Negative Correlation

Calculation of Correlation Coefficient = $\boxed{r}$

① $r = \dfrac{\text{(Covariance of A, B)}}{\sigma_A \times \sigma_B}$

② $r = \dfrac{\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}}{\sigma_x \times \sigma_y}$ or $\dfrac{\Sigma(x-\bar{x})(y-\bar{y})\,P_i}{\sigma_x \times \sigma_y}$

↓ Q without Probability ↓ Question with Probability

③ $\boxed{r_{(A,B)} = r_{(A,\,\text{Market})} \times r_{(B,\,\text{Market})}}$

Concept ④ How to Calculate Covariance

→ Covariance explains co-movement between two securities

Positive Covariance	Zero Covariance	Negative Covariance
If $x\uparrow, y\uparrow$	$x\uparrow, y =$	$x\uparrow, y\downarrow$
$x\downarrow, y\downarrow$	$x\downarrow, y =$	$x\downarrow, y\uparrow$

Calculation of Covariance

Past data (without Probability)

x	y	$(x-\bar{x})(y-\bar{y})$
✓	✓	✓
✓	✓	✓
✓	✓	✓

Step① Calculate Mean

$$\boxed{\bar{x} = \frac{\Sigma x}{n}, \quad \bar{y} = \frac{\Sigma y}{n}}$$

Step②
Calculate $(x-\bar{x}).(y-\bar{y})$

Step③
$$\text{Covariance} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{N}$$

①

Future Data (with Probability)

x	y	P_i	x_iP_i	y_iP_i	$(x-\bar{x})(y-\bar{y})P_i$
✓	✓	✓			
✓	✓	✓			
✓	✓	✓			

Step① Calculate Mean.

$$\bar{x} = \frac{\Sigma x_iP_i}{1}, \quad \bar{y} = \frac{\Sigma y_iP_i}{1}$$

Step② Calculate $(x-\bar{x})(y-\bar{y})P_i$

Step③
$$\text{Covariance} = \Sigma(x-\bar{x})(y-\bar{y})(P_i)$$

②

③ $$\text{Covariance}(A,B) = r_{(A,B)} \times \sigma_A \times \sigma_B$$

④ $$\text{Covariance}(A,B) = \beta_A \times \beta_B \times (\sigma_{Mkt})^2$$

Concept (5) Return of Portfolio

Security	(i) Return	Investment	(ii) weights	(i) x (ii)
Reliance	12%	a	a/Total	✓
Tata	14%	b	b/Total	✓
ICICI	9%	c	c/Total	✓
Yesbank	2%	d	d/Total	✓
		Total		Σ Return x weights

Σ Return x weights → Portfolio Return

Portfolio Return = Σ Return x weights

$$= \Sigma (R_A W_A + R_B \times W_B + \cdots + R_n \times W_n)$$

General Knowledge

Risk of Portfolio

Markowitz

Sharpe.

$$(A+B)^2 = A^2 + B^2 + 2AB$$

$$(A+B+C)^2 = A^2 + B^2 + C^2 + 2AB + 2BC + 2CA$$

$$\sigma_P = \sqrt{(\sigma_A W_A)^2 + (\sigma_B W_B)^2 + 2(\sigma_A W_A)(\sigma_B W_B) \times r_{(AB)}}$$

$$= \sqrt{(\sigma_A W_A)^2 + (\sigma_B W_B)^2 + 2 W_A W_B \times \text{covariance}(A,B)}$$

Concept ⑥ Calculation of Risk of Portfolio By Markowitz

ⓐ Calculation of Risk (σ_P) for Two asset model.

(when we have only two Securities in Portfolio)

$$\sigma_P = \sqrt{(\sigma_A w_A)^2 + (\sigma_B w_B)^2 + 2(\sigma_A w_A)(\sigma_B w_B) \times r_{(A,B)}}$$

or

$$= \sqrt{(\sigma_A w_A)^2 + (\sigma_B w_B)^2 + 2 w_A w_B \times \text{Covariance}(A,B)}$$

ⓑ Calculation of Risk (σ_P) for Three Asset model.

$$\sigma_P = \sqrt{(\sigma_A w_A)^2 + (\sigma_B w_B)^2 + (\sigma_C w_C)^2 + 2(\sigma_A w_A)(\sigma_B w_B)\, r_{(A,B)} + 2(\sigma_B w_B)(\sigma_C w_C)\, r_{(B,C)} + 2(\sigma_C w_C)(\sigma_A w_A)\, r_{(C,A)}}$$

ⓒ Calculation of Risk (σ_P) when we have 'n securities' with equal weights and equal standard standard.

Let number of securities = n

Equally weighted = w,

Equal standard deviation = SD = σ_S

$$\sigma_P = \sqrt{(\sigma_S)^2 \times \frac{1}{n} + r \times (\sigma_S)^2 \times \frac{(n-1)}{n}}$$

Concept (7) Portfolio with Two Securities.

Normal case

Security	weight	Return
A	w_A	R_A
B	w_B	R_B

① Return of Portfolio

$R_P = R_A w_A + R_B w_B$

② $\sigma_P = \sqrt{(\sigma_A w_A)^2 + (\sigma_B w_B)^2 + 2(\sigma_A w_A)(\sigma_B w_B) r_{(A,B)}}$

If one of security is a Risk free Asset

Security	weight	Return	SD
A	w_A	R_A	σ_A
Risk free Asset	w_B	R_f	0

① Return of Portfolio

$R_P = R_A w_A + R_f \times w_B$

② $\sigma_P = \sigma_A w_A$

Two Security Model with Borrowing.

Security	weight	Return	SD
Investment	1.2	R_A	σ_A
Borrowing	-0.2	Int rate	0

① Return of Portfolio

$= R_A w_A + R_B \times w_B$

$= R_A \times 1.2 + R_B(-0.2)$

② $\sigma_P = \sigma_A w_A$

Concept (8) find optimum Portfolio weights For Two Asset Model.

(Minimum Risk Portfolio)

Optimum Portfolio → To get high returns with low risk.

Q

Security	Return	SD
A	R_A	σ_A
B	R_B	σ_B

Covariance (A, B)

To find W_A & W_B

Sol: Step 1

$$W_A = \frac{(\sigma_B)^2 - \text{Covariance}(A,B)}{(\sigma_A)^2 + (\sigma_B)^2 - 2\,\text{Covariance}(A,B)}$$

Step 2

$$W_B = 1 - W_A.$$

Step 3

Portfolio Return =
Portfolio σ =

use Concept (7)

Concept (9) How to Calculate Beta

β = It measures sensitivity of a security price with change in market

(a) $Beta = \dfrac{\text{Change in Security Return}}{\text{Change in market Return}}$

(b) $Beta = \dfrac{\text{Covariance of Security \& MKT}}{\text{Covariance of MKT \& MKT}} = \dfrac{Cov(x,m)}{Cov(m,m)}$

(c) $Beta = \dfrac{\text{Covariance of Security \& MKT}}{\text{Variance of MKT}} = \dfrac{Cov(x,m)}{(\sigma_m)^2}$

(d) $Beta = \dfrac{r(s,m) \times \sigma_s \times \cancel{\sigma_m}}{(\sigma_m)^{\cancel{2}}} = \dfrac{r(s,m) \times \sigma_s}{\sigma_m}$

(e) Beta Calculation by Regression equation.

$Beta = \dfrac{\Sigma xy - n.\bar{x}.\bar{y}}{\Sigma y^2 - n(\bar{y})^2}$ or $\dfrac{\Sigma xM - n.\bar{x}\bar{m}}{\Sigma M^2 - n.(\bar{M})^2}$

(f) $Beta = \dfrac{\Sigma(x-\bar{x})(M-\bar{M})}{\Sigma(M-\bar{M})^2}$

Concept (10) Decision Making

(1) If Expected Return > Actual Return, Portfolio is overvalued $\boxed{\text{Sell}}$

(2) If Expected Return < Actual Return, Portfolio is undervalued $\boxed{\text{Buy}}$

Note (1)

Covariance of x with x = Variance of x

$$Cov(x, x) = (\sigma x)^2$$

Note (2) Correlation of x with x = 1

$$r_{(x, x)} = 1$$

Concept (11) Correlation & Covariance Matrix

I Covariance Matrix

Covariance	X	Y	Z
X	Variance of X $(\sigma_x)^2$	$Cov(x,y)$	$Cov(x,z)$
Y	$Cov(y,x)$	Variance of Y $(\sigma_y)^2$	$Cov(y,z)$
Z	$Cov(z,x)$	$Cov(z,y)$	Variance of Z $(\sigma_z)^2$

$Cov(x,y) = Cov(y,x)$

$Cov(x,z) = Cov(z,x)$

$Cov(y,z) = Cov(z,y)$

$Cov(x,x) =$ Variance of X

$Cov(y,y) =$ Variance of Y

$Cov(z,z) =$ Variance of Z

II Correlation Matrix

Correlation	X	Y	Z
X	1	$r(x,y)$	$r(x,z)$
Y	$r(y,x)$	1	$r(y,z)$
Z	$r(z,x)$	$r(z,y)$	1

$r(x,y) = r(y,x)$

$r(x,z) = r(z,x)$

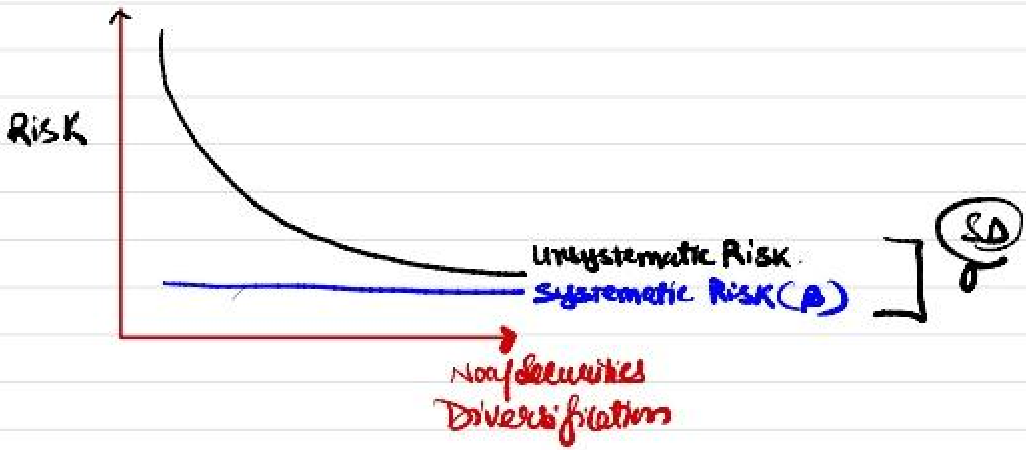
$r(y,z) = r(z,y)$

$r(x,x) = 1$

$r(y,y) = 1$

$r(z,z) = 1$

Concept 12 Types of Risk in Investing



Systematic Risk

→ It cannot be diversified away.

→ Represented by β

Unsystematic Risk

→ It can be diversified away.

Represented = Total Risk - Systematic Risk
Unsystematic Risk

Name → Random error
→ Error terms
→ Residual Risk

Concept (13) Sharpe Ratio, Treynor's & Jensen's Alpha

① **Sharpe Ratio** $= \dfrac{R_P - R_F}{\sigma_P} = \dfrac{\text{Excess over Risk free Return.}}{\text{Total Risk.}}$

- Higher is better.

② **Treynor's Ratio** $= \dfrac{R_P - R_F}{\beta_P} = \dfrac{\text{Excess over Risk free Return}}{\text{Systematic Risk.}}$

- Higher is Better.

③ **Jensen's Alpha** $\rightarrow R_P$ $-$ Expected Return as per CAPM

$\Rightarrow R_P - (R_f + \beta_P(ER_m - R_f))$

- If Alpha is positive $\rightarrow$ Better.

Concept (14) Arbitrage Pricing Theory (APT)
⇒ Stephen Ross Model.

APT suggest that CAPM is not fully explained.
(as CAPM does not include multiple factors)

⇒ APT includes multiple factors impact on expected return.

$$\text{Expected Return} = \text{Risk free rate} + \beta_{inflation} \times (\text{Factor Risk Premium})$$
$$+ \beta_{GDP} \times (\text{Factor Risk Premium})$$
$$+ \beta_{Int\,rate} \times (\text{Factor Risk Premium})$$
$$+ \beta_{Market\,Index} \times (\text{Factor Risk Premium})$$

etc.

Concept (15) Calculation of Systematic & Unsystematic Risk

(a) Sharpe model for Individual Security
(b) Sharpe model for a Portfolio
(c) Sharpe model when total risk is not found from question.
(use Reverse working)
(d) Coefficient of Determination method.

(a) Sharpe model for Individual Security.

Step ① Total Risk of Security = $\boxed{TR_S = \sigma_S^2}$ (variance of Security)

Systematic Risk

Step ②

$SR_S = (\beta_S)^2 \times (\sigma_{MKT})^2$

$= \begin{pmatrix}\text{Beta of}\\ \text{Security}\end{pmatrix}^2 \times \begin{pmatrix}\text{Variance}\\ \text{of}\\ \text{Market}\end{pmatrix}$

Unsystematic Risk

Step ③

Unsystematic Risk $= TR_S - SR_S$

(Residual risk) Step① − Step②

(b) Sharpe's Model for Portfolio

Step ① Total Risk of Portfolio = $\boxed{TR_P = (\sigma_P)^2}$ = Variance of Portfolio

Systematic Risk

Step ②

$SR_P = (\beta_P)^2 \times (\sigma_{MKT})^2$

$\begin{pmatrix}\text{Beta of}\\ \text{Portfolio}\end{pmatrix}^2 \times$ Variance of MKT

Unsystematic Risk

Step ③

$USR_P = TR_P - SR_P$

(C) Sharpe Index Model where total Risk is not given and not directly calculatable.

Step① Calculate Systematic Risk p

$SRp = (\beta p)^2 \times (\sigma_{mkt})^2$

Step② Calculate Unsystematic Risk of Portfolio.

Security	(USRs)
A	$(\sigma e)^2 \times (wA)^2 = ✓$
B	$(\sigma e)^2 \times (wB)^2 = ✓$
C	$(\sigma e)^2 \times (wC)^2 = ✓$
D	$(\sigma e)^2 \times (wD)^2 = ✓$

Unsystematic Risk of Portfolio $= USRp$

Step③ Total Risk p = SRp + USRp

(D) Finding Total Risk using Coefficient of determination (r^2)

Total Risk

Systematic Risk

$SR = TR \times r^2$

Risk which can be explained (Can be determined)

Unsystematic Risk

$USR = TR \times (1-r^2)$

- Risk which cannot be explained
- Cannot be determined.

Concept (16) Portfolio Rebalancing.

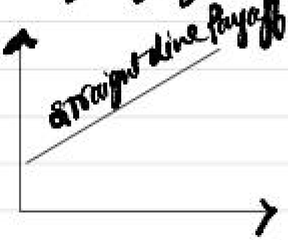
We try to maintain desired ratios of debt & equity investment at regular interval of time.

Portfolio Rebalancing / Management

Method (1)	Method (2)	Method (3) Q24, Q29
Buy & Hold Policy	**Constant Mix Policy**	**Constant Proportion Portfolio Insurance Policy (CPPI)**

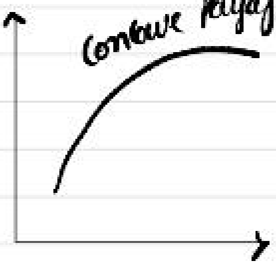
Method (1): Buy & Hold Policy

→ Do nothing
→ Buy & forget.



Method (2): Constant Mix Policy

On Rebalancing date we will rebalance the new total value in our desired ratio



Method (3): Constant Proportion Portfolio Insurance Policy (CPPI) Q24, Q29

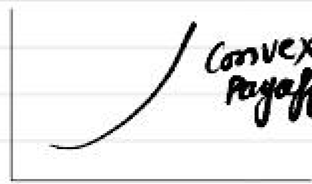
Step (1) Initially / on Rebalancing date

$$V_E = \left(\text{Total Portfolio Value} - \text{Floor Value}\right) \times m = ✓$$

Step (2)

$$V_D = \text{Balancing figure} = ✓$$

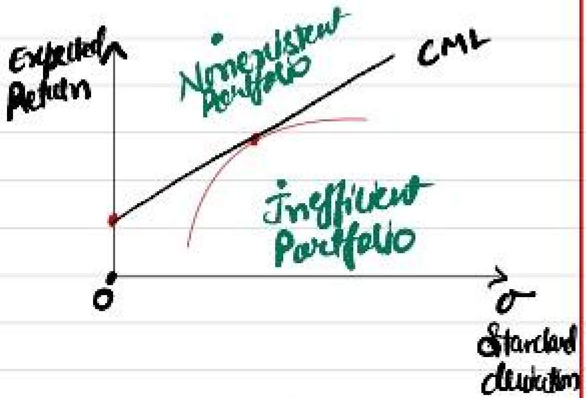
Total Portfolio Value = ★



Concept (17) CML vs SML

Capital Market Line (CML)

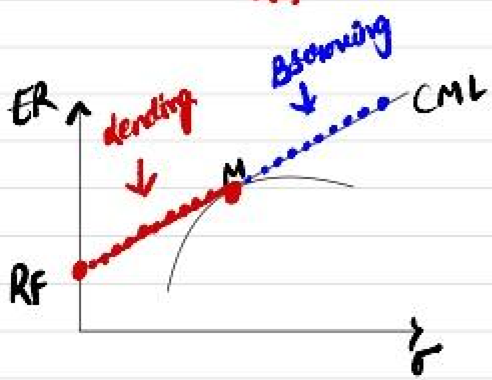
→ Based on market



Interlept = R_F

Equation ⇒

$$ER_P = R_F + \frac{\sigma_P}{\sigma_M}(ER_M - R_F)$$

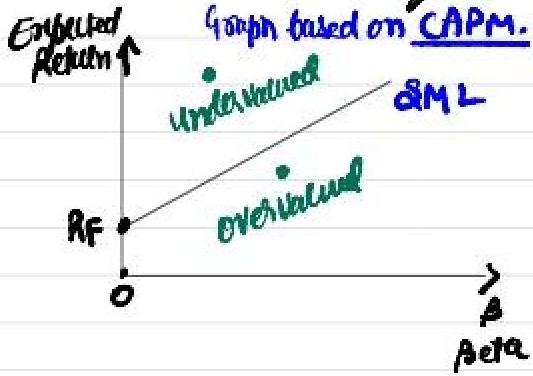


Lending Portfolio → It includes amount invested in RF Asset So money is lent out

Borrowing Portfolio → we borrow at RF rate and invest in owe

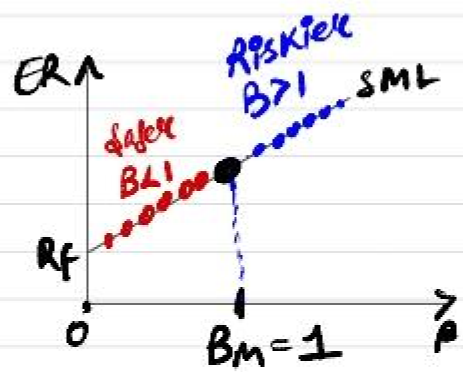
Security Market Line

→ Based on a Security



Interlept ⇒ R_F

$$ER_S = R_F + \beta_S(ER_m - R_F)$$



Portfolio.

Slope of CML ⇒

$$\frac{(ERm - RF)}{\sigma_M}$$

Slope of SML ⇒

(Market Risk Premium)

$$(ERm - RF)$$